Worksheet 8

Date: 11/01/2021

Name:

Integers and Division Algorithm

DEFINITION 1. let $a, b \in \mathbb{Z}$. We say a **divides** b if

$$b = a \cdot c \text{ for some integer} c.$$

When a divides b, we write $a|b$. Otherwise, $a \nmid b$.

Exercise

1. Label each of the following *true* or *false*, and justify your answer.

   (a) $8|0$

   (b) $a|b$ and $b|c \Rightarrow a|c$

   (c) $a|b$ and $a|c \Rightarrow a|bc$

   (d) $a|b \Rightarrow -a|b$

   (e) $a|bc \Rightarrow b|c$ or $c|a$

2. Let a and b be non zero integers

   (a) If $a|b$ and $b|a$, then $a = \pm b$.

   (b) If $a|b$, then $|a| \leq |b|$.

THEOREM 1 (The Division Algorithm)**.** *For positive integers a and b, there exist unique integers q and r such that*

$$b = aq + r \quad 0 \leq r < a$$

Recall: **highest common factor**, $hcf(a,b)$, is the largest positive integer that divides both a and b. We write this as $\gcd(a,b)$ or simply (a,b). And by we, I mean "I".
Some basic examples of this definition: $\gcd(4,2) =$
$\gcd(7,53) =$

PROPOSITION 2. *Let a and b be positive integers. If $b = aq + r$ for some integers q and r, then* $\gcd(a,b) = \gcd(r,a)$.

1. Define a **prime triple** to be a set of three prime numbers of the form $\{n, n+2, n+4\}$. For example, $\{3, 5, 7\}$ is a prime triple. Are there any others? Either exhibit another or prove there are none.

2. If a is an integer, then a^2 has a remainder of zero or one when divided by 4.